Nucleon-Nucleon Scattering Calculations in a Pole Regularized Spinor Theory *

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A pole regularized nonlinear spinor theory may be a suitable test object to compare scattering calculations of Stumpf's functional quantum theory with LSZ-results. To apply the LSZ-technique in this theory, a dressing of the occurring massless Green's function is necessary. It is shown which special approximations allow for this dressing. The renormalized nucleon-nucleon coupling constant yields the right order of magnitude for the elastic nucleon cross section.

I. Introduction

The aim of functional quantum theory is the calculation of scattering cross-sections from a fundamental theory. Each elementary particle is virtually build up from all the others, therefore one has to calculate scattering of composite particles. All methods used so far for explicite calculations start with an assumption: to each asymptotically in- or outgoing particle corresponds a local interpolating field which describes the particle during the interaction. In a unified theory of elementary particles – for instance in Heisenberg's nonlinear spinor theory [1] – the particles correspond to eigensolutions of the field equation, and one should not expect that a description of all those eigensolutions will be possible in terms of local field operators. Therefore one needs a more general scattering theory, which leads to the construction of the scattering matrix for processes between all eigensolution of the field equation. The functional quantum theory as developed by Stumpf and coworkers [2, 3] yields a procedure for the calculation of this general S-matrix. One has to calculate the scattering functionals for the ingoing and outgoing particle configurations. A scalar product of these scattering functionals determines the S-matrix element between the configurations considered. Such a functional scalar product has been proposed by Stumpf [4]. Work is in progress to clarify the mathematical properties of this scalar product [5] and to test it in scattering calculations [6].

The best test for the functional scalar product should be the direct comparison of calculated cross-sections with the measured ones. But in order to do this, one has to know a fundamental theory which describes all processes between elementary particles correctly. We all know that such a theory does not exist today. One only has some model theories that hopefully describe some aspects of the real world – one example is Heisenberg's spinor theory. Therefore, one is only able to test the scalar product together with a model theory. Moreover, for explicite calculations one has to use approximations, and nobody knows whether these "approximations" converge at al, or whether the first steps used so far give answers that are near the correct solutions. Therefore such test calculations necessarily give only very rough approximations to the experimental data. How shall we decide now, where the difference stems from? From the poor approximation used, from to great simplifications in the model theory, or finally from the functional scalar product that we want to test? This testing procedure is therefore very difficult and in this paper an other testing method is proposed, using the conventional scheme of S-matrix calculation. For this scheme is undoubtedly correct in the case of quantum electrodynamics (QED). There the LSZ-reduction formula yields results that agree with the experimental values within the extremely small experimental errors.

Every other method for the calculation of the S-matrix has to reproduce these correct results. But the QED is only of limited use as a model of a unified theory of elementary particles. It has a too simple structure: In QED there are no composite particles, and to both asymptotic particles – electron and photon – there are corresponding local

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interpolating fields that describe the particles during the interaction. Our aim is the calculation of the S-matrix for composite particles, and one has to admit instable particles, because most of the so-called elementary particles have very short life times.

For the proposed test of the functional scalar product one needs a relativistic field theory with the following properties:

1. There are bound states and resonances.
2. There are particles, which can be described by interpolating fields during the interaction.
3. The sector of the S-matrix containing only particles of typ 2 can be calculated with the LSZ-reduction technique.

Given such a theory, one is able to compare the results of LSZ-technique and functional quantum theory in the indicated sector of the S-matrix and to calculate the whole S-matrix with the new method.

The QED does not fulfil 1, we cannot use it for our test. Heisenberg's spinor theory does not fulfil 3 and we cannot use it either. The reason is the following: To apply the LSZ-reduction formula, one has to know the suitable n-point-function, the vacuum expectation value of the time-ordered product of n field operators. In a canonical theory one has inhomogeneous equations for these n-point-functions and the inhomogeneous terms are proportional to ϱ_0, the zero momentum of the spectral function of the propagator. ϱ_0 is the factor in front of the δ^3-function of the equal-time commutator. The most simple example is Eq. (34) of this paper. If one tries to solve this equations one detects the well known divergences. To avoid these infinities in Heisenberg's spinor theory, the propagator F is regularized by a dipole, leading to an indefinite metric and $\varrho_0 = 0$. The equations for the n-point-functions become homogeneous and are not sufficient to calculate the n-point-functions. This is a very serious difficulty in Heisenberg's dipole theory and one is not able to calculate crossections with the LSZ-technique. There are calculations of coupling constants within this framework [7], but they are achieved only by an unjustified replacement of $\varrho_0 G$ by F.

One theory with the required properties 1, 2, 3 is perhaps a modified spinor theory [8] (We refer to this paper as I). The dipole regularization is replaced by a weaker pole regularization. In order to keep the theory finite, a differential equation of third order is proposed. In I boundstates are calculated in a low approximation, therefore the pole theory may have property 1. The results of this paper will show that it may have the properties 2 and 3: A simple cross section will be calculated explicitely. The fundamental field η' itself cannot serve as the interpolating field required in the LSZ-formulation, for it obeys noncanonical commutation relations. But its covariant Weyl derivative χ is a canonical field. The corresponding propagator has a pole at the nucleon mass, therefore we interpret χ as interpolating field for nucleons. Now one is able to use the LSZ-technique to calculate scattering processes with only nucleons in- and outgoing. In this paper we will treat elastic nucleon-nucleon scattering. Without an adjustable parameter one gets the right order of magnitude for the cross section.

But one has to modify the approximation method used until now in the nonlinear spinor theory. In the functional equations for the τ- or φ-functional one has besides the propagator F the Green's function G from the inversion of the differential operator. Particle masses are to be calculated out of the theory, thus G has no mass term and G is a mass zero Green's function. One has to dress this G to a finite mass in order to get a nonvanishing S-matrix, therefore one needs an approximation scheme yielding this mass dressing. It will be shown how such a dressing can be achieved. This dressing has little influence on the value of the coupling constant, whereas the boson solutions are drastically changed [9]. One should calculate higher approximations to clarify the situation, before making statements about boson coupling constants, but in principle no difficulties should arise. For the corresponding Bethe-Salpeter equation has an inhomogeneous term in the pole theory, and the replacement $\varrho_0 G \to F$ can be achieved by the dressing discussed in Chapter 3.

In this paper we will restrict the calculation to the lowest approximations. To carry out the proposed test, of course one has to calculate higher approximations with the LSZ-technique parallel to corresponding calculations with the functional scalar product. Both methods should yield the same results for nucleon-nucleon scattering.

2. Asymptotic Condition and Normalization

2.1. The Pole Spinor Theory

The pole spinor theory [8] is a dynamically modified version of Heisenberg's nonlinear spinor theory [1]. Heisenberg's theory is regularized by a dipole of mass zero, whereas the pole theory is regularized by a pole only. To get a finite theory a differential equation of third order for the fundamental field is assumed. The theory is defined by a functional equation for the τ-functional

$$D_{\alpha\beta}\,\partial_\beta\,|\,\mathfrak{T}(j)\,\rangle = \{i\,\varrho_0\,j_\alpha - V_{\alpha\beta\gamma\delta}(\partial_\beta\,\partial_\gamma\,\partial_\delta + 3\,F_{\beta\gamma}\,\partial_\delta)\}\,|\,\mathfrak{T}(j)\,\rangle\,. \tag{1}$$

$|\,\mathfrak{T}(j)\,\rangle$ is the generating functional for the τ-functions, the time-ordered products of field operators between the states considered

$$|\,\mathfrak{T}(j)\,\rangle := \sum_{n=0}^{\infty} \frac{i^n}{n!}\;\tau_{\alpha_1\ldots\alpha_n}\,|\,j_{\alpha_1}\ldots j_{\alpha_n}\rangle\,. \tag{2}$$

We include the co-ordinates in the index, repeated indices hence indicate a summation over spinor indices and a 4-dimensional integration over the corresponding co-ordinates.

The field operator ψ is a hermitean Weyl spinor with spin 1/2 and isospin 1/2 [8, 10]. The Vertex V is local in configuration space and antisymmetric in all indices

$$V_{\alpha\beta\gamma\delta} = \frac{K^2}{2}\,\delta(x_\alpha - x_\beta)\,\delta(x_\alpha - x_\gamma)\,\delta(x_\alpha - x_\delta)\,\overline{V}_{\alpha\beta\gamma\delta}\,. \tag{3}$$

$\overline{V}$ is the essentially unique antisymmetrical four fermion Vertex for Weyl spinors [10, 11]. D is a differential operator of third order, with the inverse

$$G_{\alpha\beta} := i\,D^{-1}_{\alpha\beta} = i\int \frac{\mathrm{d}^4p}{(2\pi)^4}\;\frac{\overline{\Gamma}^{\nu}_{\alpha\beta}\,p_\nu}{[p^2+i\varepsilon]^2}\;\exp\{-i(p,\,x_\alpha - x_\beta)\}\,. \tag{4}$$

F is the vacuum two-point function or propagator

$$F_{\alpha\beta} := \langle 0\,|\,T\,\psi_\alpha\,\psi_\beta\,|\,0\rangle = \int dm^2\,\varrho(m^2)\,i\int \frac{\mathrm{d}^4p}{(2\pi)^4}\;\frac{\overline{\Gamma}^{\nu}_{\mu\beta}\,p_\nu}{[p^2+i\varepsilon]\,[p^2-m^2+i\varepsilon]}\;\exp\{-i(p,\,x_\alpha - x_\beta)\}\,. \tag{5}$$

Approximation methods are based on the φ-functional

$$|\,\Phi(j)\,\rangle := \exp\{\tfrac{1}{2}\,j_\alpha\,F_{\alpha\beta}\,j_\beta\}\,|\,\mathfrak{T}(j)\,\rangle = \sum_{n=0}^{\infty}\frac{i^n}{n!}\;\varphi_{\alpha_1\ldots\alpha_n}\,|\,j_{\alpha_1}\ldots j_{\alpha_n}\rangle \tag{6}$$

and the corresponding functional equation

$$\partial_\alpha\,|\,\Phi(j)\,\rangle = \{(\varrho_0\,G_{\alpha\beta} - F_{\alpha\beta})\,j_\beta + i\,G_{\alpha\alpha'}\,V_{\alpha'\beta\gamma\delta}\cdot(d_\beta\,d_\gamma\,d_\delta + 3\,F_{\beta\gamma}\,d_\delta)\}\;|\,\Phi(j)\,\rangle\,,\; d_\alpha := \partial_\alpha + F_{\alpha\beta}\,j_\beta\,. \tag{7}$$

For more details see paper I [8].

2.2. Asymptotic Condition

The functional equation (1) or (6) corresponds to the field equation

$$D_{\alpha\beta}\,\psi_\beta = V_{\alpha\beta\gamma\delta}[\psi_\alpha\,\psi_\beta\,\psi_\gamma - 3\,F_{\beta\gamma}\,\psi_\delta] \tag{8}$$

for the fundamental field ψ together with the equal time commutator

$$\{\psi_\alpha,\,\psi_\beta\}_{t_\alpha\approx t_\beta} \approx -\tfrac{1}{2}\,(t_\alpha - t_\beta)^2\;\varrho_0\,\delta_{\alpha\beta}\,\delta^3(x_\alpha - x_\beta) \Rightarrow 0\,. \tag{9}$$

This field ψ therefore cannot serve as an interpolating field for interacting particles.

But as described in I (appendix 1) the Weyl derivative χ satisfies canonical commutation relations

$$\chi_\alpha := \Gamma^{\nu}_{\alpha\beta}\,\delta^4(x_\alpha - x_\beta)\;\frac{\delta}{\delta x_{\beta}{}^{\nu}}\;\psi_\beta \tag{10}$$

$$\{\chi_\alpha, \chi_\beta\}_{t_\alpha=t_\beta} = \varrho_0\, \delta_{\alpha\beta}\, \delta^3(x_\alpha - x_\beta) \tag{11}$$

and it has a canonical propagator

$$F^c_{\alpha\beta} := \langle 0 | T\, \chi_\alpha \chi_\beta | 0 \rangle = \int \mathrm{d}m^2\, \varrho(m^2)\, i \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \frac{\Gamma^\nu_{\alpha\beta}\, p_\nu}{p^2 - m^2 + i\varepsilon} \exp\{-i(p, x_\alpha - x_\beta)\}\,. \tag{12}$$

Therefore we assume that χ is an interpolating field for "physical" particles.

Of course we cannot hope to describe real nucleons, because we are dealing with a two-component Weyl field that is not able to describe parity. Therefore one has either to double the degrees of freedom and describe the theory with Dirac spinors, thereby getting 5 possible interaction terms, or one has to introduce parity in an other way as proposed by Dürr [11]. Our structured considerations do not depend on this grouptheoretical problems and for simplicity reasons we continue with the Weyl spinors ψ and χ. Therefore our "nucleons" have no definite parity. They may be regarded as parity doubletts and our theory is completely parity degenerate. For more details concerning this problem see e. g. [12].

If we define asymptotic fields for the field χ by

$$\sqrt{Z}\, \overset{\text{in}}{\chi}{}^{\text{out}}_{\varkappa}(x) := \chi_\alpha - \int \mathrm{d}^4 y\, \overset{\text{ret}}{\triangle}{}^{\text{adv}}_{\varkappa}(x-y)\, (\Box_y + \varkappa^2)\, \chi_\alpha(y) \tag{13}$$

$$(\Box_x + \varkappa^2)\, \overset{\text{ret}}{\triangle}{}^{\text{adv}}_{\varkappa}(x) = \delta^4(x) \qquad \overset{\text{ret}}{\triangle}{}^{\text{adv}}_{\varkappa}(x) = 0 \quad \text{for} \quad x_0 \lesseqgtr 0 \tag{14}$$

we obtain as asymptotic condition for χ

$$\chi_\alpha(x) \to \sqrt{Z}\, \overset{\text{in}}{\chi}{}^{\text{out}}_{\varkappa}(x) \quad \text{for} \quad x_0 \to \mp\infty \tag{15}$$

in the usual weak sense for smeared operators. The $\overset{\text{in}}{\chi}{}^{\text{out}}$ are normalized solutions of the equation

$$(\Box + \varkappa^2)\, \overset{\text{in}}{\chi}{}^{\text{out}}_{\varkappa}(x) = 0\,. \tag{16}$$

We have to use the Klein-Gordon-equation in (13) to (16), because we need massive asymptotic particles in our Weyl theory [11].

The mass value $\varkappa^2$ has to be determined out of the theory. It is well known that in canonical theories the asymptotic conditions (13) correspond to a resp. pole in the propagator.

$$\tilde{F}^c_{\alpha\beta}(p) = \int \varrho(m^2)\, \mathrm{d}m^2\, i\, \frac{\Gamma^\nu_{\alpha\beta}\, p_\nu}{p^2 - m^2} = Z\, i\, \Gamma^\nu_{\alpha\beta}\, p_\nu \left(\frac{1}{p^2 - \varkappa^2} + N^\Delta(p^2) \right). \tag{17}$$

In other words, if the spectral function ϱ has the form

$$\varrho(m^2) = Z\, \delta(m^2 - \varkappa^2) + \varrho^\Delta(m^2) \tag{18}$$

with $Z > 0$ and N^Δ resp. ϱ^Δ finite at the nucleon pole $\varkappa^2$, then the asymptotic condition (13) or (15) is valid.

This conventional description of the asymptotic states is only possible for the nucleons, because we do not have local field operators for the compound states. Furthermore we do not know at present, whether the ghost states with negative norm in (5) destroy the unitarity of the S-matrix. We ignore this serious problems here, and draw some simple consequences for the elastic nucleon scattering in the next chapters.

2.3. *Normalization of the Spinor Field*

The normalization of the field χ is not yet fixed. This may be done by fixing ϱ_0 or Z. If we change the fields by a factor d, we have to change all other quantities accordingly

$$\begin{aligned} &\psi \to d\,\psi \qquad && \chi \to d\,\varkappa \\ &F \to d^2\,\psi \qquad && K^2 \to \frac{1}{d^2} K \\ &Z \to d^2 Z \qquad && \varrho \to d^2 \varrho \quad \text{etc.} \end{aligned} \tag{19}$$

Two special normalizations are in use.

a) $d^{\mathrm{c}}=\varrho^{-\frac{1}{2}}$. This may be called the canonical normalization, because one gets the canonical commutation relations without a factor

$$\{\chi_\alpha^{\mathrm{c}}, \chi_\beta^{\mathrm{c}}\}_{t_\alpha=t_\beta}=\delta^3(x_\alpha-x_\beta)\,. \tag{20}$$

b) $d^{\mathrm{R}}=Z^{-\frac{1}{2}}$. This is called (wave-function-) renormalization. One gets the asymptotic condition without a factor

$$\chi_\alpha^{\mathrm{R}}(x)\to\overset{\text{in}}{\underset{\text{out}}{\chi_z}}(x) \quad \text{for} \quad x_0\to\mp\infty\,. \tag{21}$$

One has to compute Z^{c} and $\varrho_0{}^{\mathrm{R}}$. Because the ratio ϱ_0/Z is independent of the normalization one has

$$\varrho_0{}^{\mathrm{R}}=\varrho_0/Z=1/Z^{\mathrm{c}}\,. \tag{22}$$

In paper I the renormalized version with $Z^{\mathrm{R}}=1$ has been used. It is not necessary to fix the normalization of χ at all, because it has no physical consequences. All physical quantities are independent of d. The physical 4 nucleon coupling constant, for example, is K^2Z^2/ϱ_0 (53). In the following we will repeat the selfconsistent calculation of the two-point function without specifying the normalization.

2.4. *Calculation of the Propagator F*

In I an inhomogenous nonlinear equation for F has been derived (I.94)

$$F_{\alpha\tau}=\varrho_0 G_{\alpha\tau}+6\left(\frac{K^2}{2i}\right)^2 G_{\alpha\beta}V^i_{\beta\gamma}G_{\gamma\delta}V^j_{\delta\mu}F_{\mu\nu}V^i_{\nu\lambda}F_{\lambda\varrho}V^j_{\varrho\sigma}F_{\sigma\tau}\,. \tag{23}$$

This can be solved for F to give

$$\tilde{F}(p)=\varrho_0\,\tilde{G}(p)\left[1+\frac{3}{2}\left(\frac{K}{2\pi\varkappa}\right)^4 L(\lambda)\right]^{-1}, \quad \lambda:=p^2/\varkappa^2\,, \tag{24}$$

$$L(\lambda):=\frac{1}{3}\,(g_{\lambda\mu}g_{\mu\varrho}+g_{\lambda\nu}g_{\mu\varrho}+g_{\lambda\varrho}g_{\mu\nu})\,\frac{p^\lambda}{(p^2)^2}\,L^{\mu\nu\varrho}(p)\,,$$

$$L^{\mu\nu\varrho}(I):=\frac{\varkappa^2}{\pi^4}\int \mathrm{d}^4r\,\mathrm{d}^4s\,\tilde{G}^\mu(I-r-s)\,\tilde{F}^\nu(r)\tilde{F}^\varrho(s)\,. \tag{25}$$

We make the ansatz (18) for F and drop N^Δ in the calculation of $L^{\mu\nu\varrho}$. L is linear in the three propagators G, F and F, therefore we get a factor Z from each F and obtain

$$\tilde{F}(p)=\varrho_0\,\tilde{G}(p)\left[1+\frac{3}{2}\left(\frac{K}{2\pi\varkappa}\right)^4 Z^2\,L^{\mathrm{R}}(\lambda)\right]^{-1} \tag{26}$$

where L^{R} is the convolution with $Z=1$. This L^{R} has been calculated in I. The right hand side of (26) $\tilde{F}_{\mathrm{r.h.}}$ has a pole at $p^2=0$ and at a finite $p^2=\varkappa_1{}^2$. The selfconsistence requirement $\varkappa_1{}^2=\varkappa^2$ fixes the coupling constant

$$\left(\frac{K^2Z}{(2\pi\varkappa)^2}\right)^2=\frac{-2}{3\,L^{\mathrm{R}}(1)} \tag{27}$$
$$=-\frac{2}{3}\left[\frac{1}{3}-\frac{\sqrt{3}}{6}\pi\right]^{-1}\approx(1.078)^2\,.$$

Comparison of the residua at this pole yields

$$\frac{\varrho_0}{Z}=\frac{L^{\mathrm{R}\prime}(1)}{-L^{\mathrm{R}}(1)}=\left(1-\frac{\sqrt{3}}{3}\pi\right)\Big/\left(-\frac{1}{3}+\frac{\sqrt{3}}{6}\pi\right)$$
$$\approx 1.419\,. \tag{28}$$

Inserting (27) and (28) into (26) we obtain

$$\tilde{F}(p)=Z\,\tilde{G}(p)\,\lambda\,L^{\mathrm{R}\prime}(1)/[L^{\mathrm{R}}(\lambda)-L^{\mathrm{R}}(1)]\,. \tag{29}$$

We have to look, whether the dropping of ϱ^Δ or N^Δ in the evaluating of L is justified. Comparing (17) with (29) we get

$$\frac{1}{\lambda-1}+\bar{N}^\Delta(\lambda)=\frac{L^{\mathrm{R}\prime}(1)}{\lambda[L^{\mathrm{R}}(\lambda)-L^{\mathrm{R}}(1)]}=:\bar{N}^{\mathrm{cal}}(\lambda) \tag{30}$$

with

$$\bar{N}^{(\Delta,\,\mathrm{cal})}(\lambda):=\varkappa^2\,N^{(\Delta,\,\mathrm{cal})}(p^2)\,.$$

From Fig. 1 we learn that N^Δ is small for all p^2, thus the evaluation of L with the pole term only is sufficient in our low approximation.

We conclude that F has indeed a pole at $p^2=\varkappa^2$ with a residuum $Z>0$, therefore the asymptotic condition (13) is valid and we are allowed to interpret the spinor potential χ as interpolating field for particles of mass $\varkappa$.

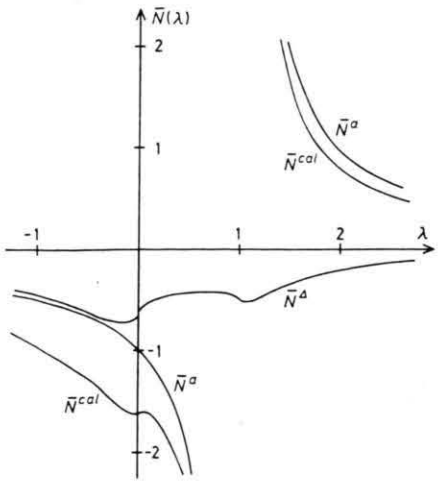


Fig. 1. Calculated pole function $\bar{N}^{\mathrm{cal}}(\lambda)$ of the fermion propagator compared with the ansatz $\bar{N}^{\mathrm{a}}(\lambda)=(\lambda-1)^{-1}$. The difference $\bar{N}^{\Delta}(\lambda)$ is small enough to justify its neglection in the convolution integral L.

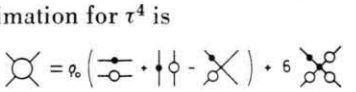
3. Dressing of the Massless Green's Function

3.1. The Dressing Problem

From the asymptotic condition (13) one derives in the well known manner the LSZ-reduction formula for the S-matrix. The S-matrix element for elastic nucleon-nucleon scattering is proportional to the amputated cavuum 4-point function on the mass shell

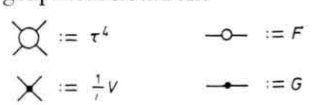
$$\langle p_1 p_2 | S | p_3 p_4 \rangle \sim \frac{1}{Z^2} \prod_{i=1}^{4} \cdot (p_i^2 - \varkappa^2)\, \tau^4(p_1 p_2 p_3 p_4) \Big|_{p_i^2=\varkappa^2}. \tag{31}$$

In our pole spinor theory one has to compute this τ^4-function from Equation (1) or (7). Then one has to face the following problem. These equations contain D or G, the massless Green's function (4). All simple approximations for τ^4 therefore yield a factor G at one coordinate at most [10]. The lowest approximation for τ^4 is



(32)

with the graphical notations [8]



(33)

To compute the S-matrix (31), one has to multiply each variable by the resp. $(p_i^2-\varkappa^2)$. This factors compensate the poles of the propagators F, but at the G the factor $(p^2-\varkappa^2)$ survives, and going on the mass shell we obtain a vanishing S-matrix.

In canonical theories there are approximation procedures avoiding this difficulty, for example functional methods using F alone [13]. Because of the vanishing first spectral moment ϱ_0 this methods are not available in theories with indefinite metric. In Heisenberg's dipole spinor theory therefore no scattering solutions are known. The pole spinor theory has indefinite metric, too, but it is possible to reformulate it as a canonical theory with a nonlocal vertex V^{c} [8, Appendix]. Therefore one should be able to dress all G-lines to F-lines to get a non-vanishing S-matrix.

3.2. The Dressing Method

The pole theory is formulated in terms of the φ-functional (6) and we want to dress the φ-functions in this formulation. We achieve the dressing of G to F by the inhomogenous equation for the propagator F. The lowest nontrivial approximation for this equation is (23), its general form is

$$F_{\alpha\beta} = \varrho_0\, G_{\alpha\beta} + G_{\alpha\gamma}\, M_{\gamma\delta}\, F_{\delta\beta} \tag{34}$$

or solved for F

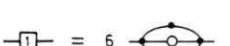
(35)

This corresponds to the general eigenvalue-equation for the fermion φ-function

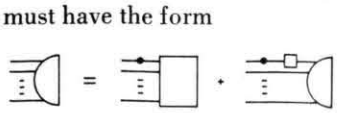
$$\varphi_\alpha^{\mathrm{F}} = G_{\alpha\gamma}\, M_{\gamma\delta}\, \varphi_\delta^{\mathrm{F}} \tag{36}$$

and F has poles at the solutions of (36). According to (23), the lowest contribution to this mass operator M is

$$M^1_{\beta\sigma} = 6\, \frac{1}{i^2}\, V_{\beta\gamma\nu\lambda}\, V_{\delta\mu\varrho\sigma}\, G_{\gamma\delta}\, F_{\mu\nu}\, F_{\lambda\varrho} \tag{37}$$

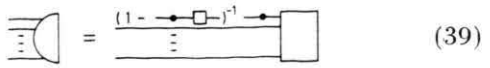


To dress a G-line to an F-line, the φ-equation considered must have the form



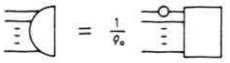
(38)

We bring the last term to the l. h. side and devide by $(1 - -\bullet\text{-}\square\text{-})$



(39)

This devision is no problem. $G_{\alpha\beta}$ and $M_{\alpha\beta}$ depend only on the co-ordinate difference $(x_\alpha - x_\beta)$, therefore $\delta_{\alpha\beta} - G_{\alpha\gamma} M_{\gamma\beta}$ is a simple factor in momentum space. Using (35) we obtain

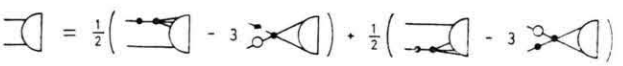


(40)

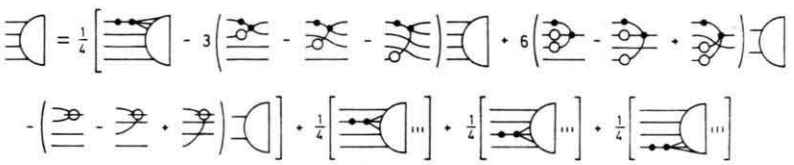
3.3. Symmetrized φ-equations

As an explicite example for this dressing mechanism I will treat the boson eigenvalue problem. But the same arguments are valid for all φ-equations, leading to the dressing of all G to F, except the one G in the dressing equation (35) itself.

To solve the φ-equation (7) for the boson problem we may use the well known NTD-procedure [1] or a more systematic functional approximation scheme [4, 8]. With the integral average of I the lowest φ-equations are



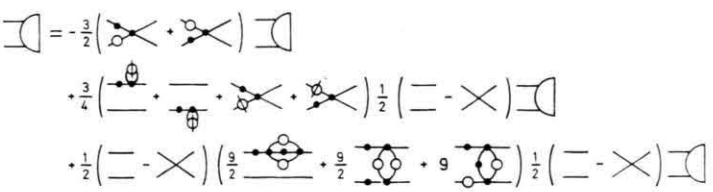
(41)



(42)

with $\text{-}()\text{-} : * \text{-}\bigcirc\text{-} * \varrho_0 \text{-}\bullet\text{-}$.

The systematic approximation method demands: Neglect φ^6 in (42), solve (42) for φ^4 and insert this φ^4 in (41), finally solve the resulting equation for φ^2 to get eigenvalues and eigensolutions. For our structural considerations we use a simpler method: Insert (42) in (41) and neglect then φ^4 and φ^6. This is still more than perturbation theory because of the exact propagator F. We obtain

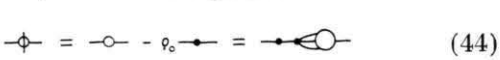


(43)

This equation does not allow for the dressing of G to F. The required selfenergy terms proportional to M^1 (37) do occur, but they carry wrong factors. Contributions from higher φ-functions do not help, because they involve higher powers of the vertex V. One may worry about the terms with

$$\text{-}()\text{-} = \text{-}\bigcirc\text{-} * \varrho_0 \text{-}\bullet\text{-},$$

but in higher approximations these $\text{-}()\text{-}$ should be replaced by the vacuum 4-point function

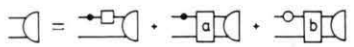


(44)

These terms do not occur, if one subtracts out not only the two-point-function F but higher vacuum-contractions as well. In any case, these terms do not change the wrong factors to their correct values. Afterwards, when we got the desired dressing, those $\text{-}()\text{-}$-terms will be compensated by approximations to the vacuum 4-point function. An example is Equation (54). Inverting with $(1 - \frac{3}{4} G M)$ would yield a nonzero mass at the leg considered, but not the nucleon mass: the S-matrix would still vanish.

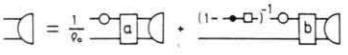
Of course one could dress the G in (43) by adding the missing terms and subtracting them

again. But that would not help, it leads to an equation of the typ



(45)

After dressing the G-line we obtain

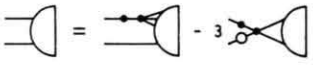


(46)

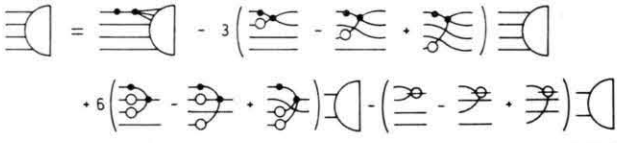
The last term has a double pole at the nucleon mass. Corresponding equations for the τ^4-function yield a diverging S-matrix, because only one pole is removed by the reduction formula (31).

4.4. *Unsymmetrical φ-equations*

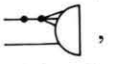
To achieve the necessary dressing we have to abandon the symmetrized φ-Eqs. (41), (42) and have to go back to the original unsymmetrical φ-equation [10].



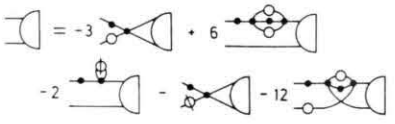
(47)



(48)

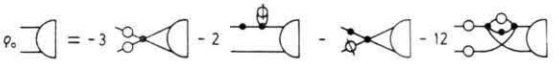
In these φ-equations one coordinate is singled out and we have four possibilities to insert (48) into (47). This corresponds to the four field operators in the resp. τ^4-function, each of which we may choose to replace it by $G\,V\,\psi^3$ (1). In other words, we may iterate each of the four legs of  , thereby producing a G-line at the leg considered. One motive for the use of the symmetrical equations (41), (42) was the removal of this arbitrariness in the approximation scheme. Now we use this freedom to achieve the dressing of G.

The Equations (38) to (40) show, that we have to produce the complete massoperator M at the leg, the G-function is standing at. Therefore we have to iterate in exactly the same manner, we do in deriving M itself. We conclude, that we must iterate the inner legs of  that go the vertex and not the outer leg, because in the Eq. (36) for M there is no outer leg. Which of the three inner legs we choose is irrelevant because of the antisymmetry of the vertex V. In our example we have to insert (48) into (47) with the G-line of (48) at one of the three upper legs of φ^4 in (47) and we obtain



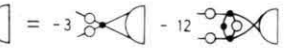
(49)

Now the mass term has the right factor 6 and we obtain after the dressing



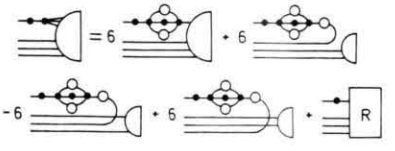
(50)

If we neglect the vacuum 4-point-function  (44), we end up with a symmetrical equation in spite of the unsymmetrical φ-equations we started with.



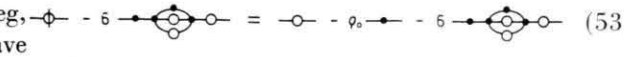
(51)

If we wish to dress the inner G-line of the last term in (50), we have to consider the 6-point function in (48). This will lead to the dressing of the G-lines in (48). The last 3 terms in (48) have a factor F at the upper leg, leading to a double pole after the dressing. But this terms will be cancelled automatically. We have to iterate one of the upper inner legs of the 6-point function. Writing down all occuring mass terms explicitely we obtain



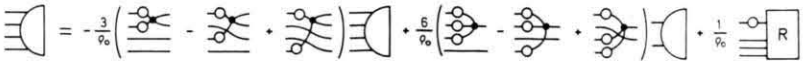
(52)

The rest R contains no mass terms. Inserting this into (48), we get in the last three terms of (48)



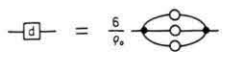
(53)

This vanishes in the approximation we use for our dressing (34), (37) and we obtain



(54)

Inserting this into (47) we observe, that we have to dress the G-line there with an improved dressed operator M^{d}.



The corresponding dressed fermion eigenvalue equation will be discussed in an other paper [9].

3.5. Conclusion

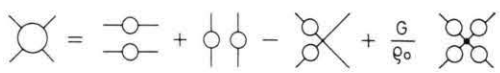
To dress all inner G-lines, we simply have to iterate always the innermost leg, thereby producing G-chains without any branching. That is, we have to choose the most unsymmetrical iteration to achieve the necessary dressing. Numerical calculations in the dipole spinor theory and with the anharmonic oszillator have shown, too, that one has to prefer unsymmetrical iterations of the φ-equations. They yield better eigenvalues than symmetrical iterations [14].

The use of the symmetrical Eqs. (41), (42) is suggested by functional consideration. The exact n-point functions are (anti-)symmetrical in all their variables and every functional solving procedure for the τ- or φ-functional itself would yield symmetrical results at each step of the calculation. But we have seen that the symmetrical φ-equations are not suitable for the calculation of scattering amplitudes with the LSZ-reduction technique. One has to use the unsymmetrical equations to get the necessary dressing. The result is a symmetrical equation, but intermediate steps have to deal with unsymmetrical objects. Our approximations are based on a power series expansions of the functionals. This expansions are truncated and one does not treat the functionals themselves. We conclude that such formal functional requirements as total antisymmetry are not always the best tool for handling the expansion coefficients.

In other sectors of the S-matrix with outgoing bound states one has to face the same problem. To achieve the correct dressing of the φ-functions for the outgoing particles, one has to deal with unsymmetrical objects, one has to use the φ-functions itself and not the φ-functional. In the approach of functional quantum theory one needs unsymmetrical equations at intermediate steps, too, in this case to formulate the correct boundary conditions for the scattering functionals [15].

4. Renormalized Coupling Constant

For the calculation of the S-matrix element (31) we had to dress the G-factor of the τ^4-function and we have learned to achieve this. The consequent procedure would be to dress all inner G-functions as well. But the resulting eigenvalue kernels are much more complicated. The boson kernel $F * F$ is related to the fermion kernel $G * F * F$, and can be evaluated analytically, the fermion kernel $F * F * F$, however, requires numerical integrations [9]. As a first step we therefore adopt the following approximation method: We dress only the outer G-line to get a nontrivial S-matrix, and obtain instead of (32)



(55)

Higher contributions to (55) will lead to poles of the τ^4-function at the boson eigenvalues. But before calculating them, one should consider the dressed eigenvalue kernels. This will be done in another paper [9].

In Heisenberg's dipole spinor theory ϱ_0 is zero and the dressing of (32) to (55) is not possible. For the calculation of boson-nucleon coupling constants [7] (55) has been used without the factor ϱ_0. In the pole spinor theory analogous calculations can be done without principal difficulties.

From (31) and (55) we derive the physical 4-nucleon coupling constant

$$K^2_{\mathrm{phs}} = (Z^2/\varrho_0)\, K^2 \,. \tag{56}$$

We get a factor Z^4 from the residua of the 4 propagators F in (55) and a factor Z^{-2} in the reduction formula (31), that removes the residua of the free terms $F\,F$ in (55) to yield the proper normalization of the S-matrix element.

We may write the field equation (8) itself with this physical coupling constant K^2_{phs}. To do this we define a renormalized Green's function

$$G^{\mathrm{R}}_{\alpha\beta} := (1/\varrho_0)\, G_{\alpha\beta} \,. \tag{57}$$

This G^{R} will be dressed to F without a factor ϱ_0. The normalized fields χ^{R}, that obey the asymptotic condition (15) without any factor, then satisfy the equation

$$\chi_\varepsilon{}^{\mathrm{R}} = G^{\mathrm{R}}_{\varepsilon\alpha}\left(\frac{Z^2}{\varrho_0}\, V^{\mathrm{C}}_{\alpha\beta\gamma\delta}\right)\, [\chi_\beta{}^{\mathrm{R}}\, \chi_\gamma{}^{\mathrm{R}}\, \chi_\delta{}^{\mathrm{R}} - 3\, F^{\mathrm{R}}_{\beta\gamma} \chi_\delta{}^{\mathrm{R}}] \,. \tag{58}$$

Instead of K^2 in (8) we now got the renormalized physical coupling constant K^2_{phs} again. Inserting (27) and (28) into (56) we obtain

$$\left(\frac{K_{\mathrm{phs}}}{2\pi\varkappa}\right)^2 = \sqrt{\frac{-2}{3L^{\mathrm{R}}(1)}\cdot\frac{-L^{\mathrm{R}}(1)}{L^{\mathrm{R}'}(1)}} = \frac{1.078}{1.419} = 0.760\,. \tag{59}$$

From (55) we derive with the standard methods the constant differential cross section for elastic proton-proton scattering

$$\frac{\mathrm{d}\sigma_{\mathrm{pp}}}{\mathrm{d}t} = \frac{1}{4\pi}\left(\frac{K_{\mathrm{phs}}}{\varkappa^2}\right)^2\frac{1}{\varkappa^4}\frac{(s-2\varkappa^2)^2}{s(s-4\varkappa^2)} = 35.9\,\frac{(s-2\varkappa^2)^2}{s(s-4\varkappa^2)}\,\frac{m\,b}{(\mathrm{GeV}/c)^2}\,. \tag{60}$$

Thus we got the right order of magnitude near the threshold. To obtain better results one has to use better approximations for the τ^4-function (55). All methods developed for canonical theories are applicable to the canonical form of the pole spinor theory, for example one can derive an inhomogeneous Bethe-Salpeter-equation for τ^4.

We conclude, that the LSZ-technique can be used for the calculation of cross sections in the pole theory. Therefore this theory is indeed suitable for the test of functional quantum theory.

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